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In $\triangle ABC$ the following relationship holds:

$$a^2\sqrt{\cos A} + b^2\sqrt{\cos B} + c^2\sqrt{\cos C} \leq \frac{9\sqrt{2}R^2}{2}$$

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Solution by Tapas Das-India

$$\begin{aligned} \sum \cos A &\stackrel{CBS}{\leq} \sqrt{3(\cos A + \cos B + \cos C)} = \sqrt{3\left(1 + \frac{r}{R}\right)} \stackrel{Euler}{\leq} \frac{3}{\sqrt{2}} \\ a^2\sqrt{\cos A} + b^2\sqrt{\cos B} + c^2\sqrt{\cos C} &\stackrel{Chebyshev}{\leq} \frac{1}{3}(a^2 + b^2 + c^2) \left(\sum \cos A\right) \stackrel{Leibniz}{\leq} \\ &\leq \frac{9R^2}{3} \cdot \frac{3}{\sqrt{2}} = \frac{9\sqrt{2}R^2}{2} \end{aligned}$$

Equality holds for an equilateral triangle.