

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\frac{\sin^2 A}{1 + \cos A} + \frac{\sin^2 B}{1 + \cos B} + \frac{\sin^2 C}{1 + \cos C} \geq \frac{3}{2}$$

Proposed by Sarkhan Adgozalev-Georgia

Solution by Daniel Sitaru

$$\begin{aligned} \sum_{cyc} \frac{\sin^2 A}{1 + \cos A} &= \sum_{cyc} \frac{1 - \cos^2 A}{1 + \cos A} = \sum_{cyc} (1 - \cos A) = \\ &= 3 - \sum_{cyc} \cos A \stackrel{JENSEN}{\geq} 3 - 3\cos\left(\frac{A + B + C}{3}\right) = 3 - 3\cos\frac{\pi}{3} = 3 - \frac{3}{2} = \frac{3}{2} \end{aligned}$$

Equality holds for an equilateral triangle.