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In any acute Δ ABC the following relationship holds :

$$\sqrt{\cos A}+\sqrt{\cos B}+\sqrt{\cos C}\le\sqrt{\sin\frac{A}{2}}+\sqrt{\sin\frac{B}{2}}+\sqrt{\sin\frac{C}{2}}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\sqrt{\cos A}+\sqrt{\cos B}+\sqrt{\cos C}=\frac{1}{2}\sum_{cyc}\left(\sqrt{\cos B}+\sqrt{\cos C}\right)\overset{CBS}{\le}$$

$$\le\frac{1}{2}\cdot\sum_{cyc}\left(\sqrt{2(\cos B+\cos C)}\right)=\frac{1}{\sqrt{2}}\cdot\sum_{cyc}\left(\sqrt{2\cos\frac{B+C}{2}\cdot\cos\frac{B-C}{2}}\right)$$

$$=\sum_{cyc}\sqrt{\sin\frac{A}{2}\cdot\cos\frac{B-C}{2}}\le\sum_{cyc}\sqrt{\sin\frac{A}{2}}\left(\because 0<\cos\frac{B-C}{2}\le 1\text{ and analogs}\right)$$

$$\text{and so},\sum_{cyc}\sqrt{\cos A}\le\sum_{cyc}\sqrt{\sin\frac{A}{2}}\ \forall\text{ acute }\Delta\text{ ABC},$$

$$''='' \text{ iff } \Delta\text{ ABC is equilateral (QED)}$$