ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \sin A(\cot B + \cot C) \geq 3$$

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Solution by Sarkhan Adgozalov-Georgia

$$\sum \sin A(\cot B + \cot C) = \sum \sin A\left(\frac{\cos B}{\sin B} + \frac{\cos C}{\sin C}\right) =$$

$$= \sum \left(\frac{\sin A}{\sin B} \cdot \cos B + \frac{\sin A}{\sin C} \cdot \cos C\right) = \sum \left(\frac{a}{b} \cdot \frac{a^2 + c^2 - b^2}{2ac} + \frac{a}{c} \cdot \frac{a^2 + b^2 - c^2}{2ab}\right) =$$

$$\sum \left(\frac{a^2 + c^2 - b^2}{2bc} + \frac{a^2 + b^2 - c^2}{2bc}\right) = \sum \frac{a^2}{bc} \geq \frac{(a + b + c)^2}{ab + bc + ca} \geq 3$$

$$\textit{Equality holds for}: a = b = c.$$