ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a\cos(A)+b\cos(B)+c\cos(C)}{a\sin(A)+b\sin(B)+c\sin(C)} \le \frac{\sqrt{3}}{3}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Mirsadix Muzefferov-Azerbaijan

$$a\cos(A)+b\cos(B)+c\cos(C)=\sum_{cyc} a\cos(A)=2R\sum_{cyc} \sin A\cos A=$$

$$=R\sum_{cyc}\sin 2A=4R\sin A\sin B\sin C=\frac{2F}{R}=\frac{2rs}{R}$$

$$a\sin(A)+b\sin(B)+c\sin(C)=\sum_{cyc} a\cdot\frac{a}{2R}=\frac{a^2+b^2+c^2}{2R}$$

$$\frac{a\cos(A)+b\cos(B)+c\cos(C)}{a\sin(A)+b\sin(B)+c\sin(C)}=\frac{\frac{2rs}{R}}{\frac{a^2+b^2+c^2}{2R}}=$$

$$=\frac{4rs}{a^2+b^2+c^2}\overset{IONESCU-WEITZENBOCK}{\leq}\frac{4F}{4\sqrt{3}F}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

$$\textit{Equality holds for}: a=b=c.$$