ROMANIAN MATHEMATICAL MAGAZINE

In acute ΔABC the following relationship holds:

$$\tan^{2026} A + \tan^{2026} B + \tan^{2026} C \geq \left(\cot^{2026}\frac{A}{2} + \cot^{2026}\frac{B}{2} + \cot^{2026}\frac{C}{2}\right)$$

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Solution by Tapas Das-India

$$\tan A + \tan B = \frac{sinA}{cosA} + \frac{sinB}{cosB} = \frac{\sin(A+B)}{cosAcosB} \overset{A+B+C=\pi}{=}$$

$$= \frac{sinC}{cosAcosB} = \frac{2sinC}{\cos(A+B) + \cos(A-B)} \overset{A+B+C=\pi \& \cos(A-B)\leq 1}{=}$$

$$= \frac{2sinC}{1 - \cos C} = \frac{4\sin\frac{C}{2}\cos\frac{c}{2}}{2\sin^2\left(\frac{C}{2}\right)} = 2\cot\frac{C}{2}$$

$$\tan^{2026} A + \tan^{2026} B \overset{CBS}{\geq} \frac{2}{2^{2026}}(\tan A + \tan B)^{2026} \geq$$

$$\geq \frac{2}{2^{2026}}\left(2\cot\frac{C}{2}\right)^{2026} = 2\cot^{2026}\frac{C}{2} \text{ or } \cot^{2026}\frac{C}{2} \leq \frac{1}{2}(\tan^{2026} A + \tan^{2026} B)$$

$$\left(\cot^{2026}\frac{A}{2} + \cot^{2026}\frac{B}{2} + \cot^{2026}\frac{C}{2}\right) =$$

$$= \sum \cot^{2026}\frac{C}{2} \leq \frac{1}{2}\sum(\tan^{2026} A + \tan^{2026} B) = \tan^{2026} A + \tan^{2026} B + \tan^{2026} C$$

Equality holds for an equilateral triangle.