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In ΔABC the following relationship holds:

$$\frac{a}{\sqrt{(b+c)^2-a^2}}+\frac{b}{\sqrt{(c+a)^2-b^2}}+\frac{c}{\sqrt{(a+b)^2-c^2}}\ge\sqrt{3}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\frac{a}{\sqrt{(b+c)^2-a^2}}+\frac{b}{\sqrt{(c+a)^2-b^2}}+\frac{c}{\sqrt{(a+b)^2-c^2}}=$$

$$=\sum_{cyc}\frac{a}{\sqrt{(b+c)^2-a^2}}=\sum_{cyc}\frac{a}{\sqrt{(b+c-a)(b+c+a)}}=$$

$$=\sum_{cyc}\frac{a}{\sqrt{2s(2s-2a)}}=\frac{1}{2}\sum_{cyc}\frac{a}{\sqrt{s(s-a)}}\overset{AM-GM}{\ge}\frac{1}{2}\cdot 3\sqrt[3]{\frac{abc}{s\cdot\sqrt{s(s-a)(s-b)(s-c)}}}=$$

$$=\frac{3}{2}\cdot\sqrt[3]{\frac{4RF}{s\cdot F}}=\frac{3}{2}\cdot\sqrt[3]{\frac{4R}{s}}\overset{MITRINOVICI}{\ge}\frac{3}{2}\cdot\sqrt[3]{\frac{4\cdot\frac{2}{3\sqrt{3}}\cdot s}{s}}=\frac{3}{2}\sqrt[3]{\frac{8}{3\sqrt{3}}}=\frac{3}{2}\cdot\frac{2}{\sqrt{3}}=\sqrt{3}$$

Equality holds for an equilateral triangle.