ROMANIAN MATHEMATICAL MAGAZINE

In acute ΔABC the following relationship holds:

$$\frac{a^2+b^2}{cosC}+\frac{b^2+c^2}{cosA}+\frac{c^2+a^2}{cosB}\geq 36R^2$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\frac{a^2+b^2}{cosC}+\frac{b^2+c^2}{cosA}+\frac{c^2+a^2}{cosB}=\sum_{cyc}\frac{a^2+b^2}{cosC}\overset{AM-GM}{\geq}$$

$$\geq 2\sum_{cyc}\frac{ab}{cosC}=2\sum_{cyc}\frac{\frac{2F}{sinC}}{cosC}=8F\sum_{cyc}\frac{1}{2sinCcosC}=8F\sum_{cyc}\frac{1}{sin2C}\geq$$

$$\overset{BERGSTROM}{\geq} 8F\cdot\frac{(1+1+1)^2}{sin2A+sin2B+sin2C}=\frac{72F}{4sinAsinBsinC}=$$

$$=\frac{18F}{\frac{a}{2R}\cdot\frac{b}{2R}\cdot\frac{c}{2R}}=\frac{18F\cdot 8R^3}{4RF}=36R^2$$

Equality holds for an equilateral triangle.