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In $\triangle ABC$ the following relationship holds:

$${}^{2026}\sqrt{\sin A} + {}^{2026}\sqrt{\sin B} + {}^{2026}\sqrt{\sin C} \leq \sqrt[2026]{\cos \frac{A}{2}} + \sqrt[2026]{\cos \frac{B}{2}} + \sqrt[2026]{\cos \frac{C}{2}}$$

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Solution by Tapas Das-India

$$\begin{aligned} \sin A + \sin B &= 2 \sin \frac{A+B}{2} \cdot \cos \frac{A-B}{2} \stackrel{A+B+C=\pi \text{ \& } \cos \frac{A-B}{2} \leq 1}{\leq} 2 \cos \frac{C}{2} \\ {}^{2026}\sqrt{\sin A} + {}^{2026}\sqrt{\sin B} &\stackrel{CBS}{\leq} 2 \sqrt[2026]{\left(\frac{\sin A + \sin B}{2}\right)} \leq 2 \sqrt[2026]{\left(\frac{2 \cos \frac{C}{2}}{2}\right)} = \\ &= 2 \sqrt[2026]{\cos \frac{C}{2}} \text{ then } \sqrt[2026]{\cos \frac{A}{2}} \geq \frac{1}{2} ({}^{2026}\sqrt{\sin A} + {}^{2026}\sqrt{\sin B}) \\ \sqrt[2026]{\cos \frac{A}{2}} + \sqrt[2026]{\cos \frac{B}{2}} + \sqrt[2026]{\cos \frac{C}{2}} &\geq \frac{1}{2} \sum ({}^{2026}\sqrt{\sin A} + {}^{2026}\sqrt{\sin B}) = \\ &= {}^{2026}\sqrt{\sin A} + {}^{2026}\sqrt{\sin B} + {}^{2026}\sqrt{\sin C} \end{aligned}$$

Equality holds for an equilateral triangle.