

ROMANIAN MATHEMATICAL MAGAZINE

In acute $\triangle ABC$ the following relationship holds:

$$\frac{ab}{a^2 + b^2 - c^2} + \frac{bc}{b^2 + c^2 - a^2} + \frac{ca}{c^2 + a^2 - b^2} \geq 3$$

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Solution by Daniel Sitaru-Romania

$$\begin{aligned} \frac{ab}{a^2 + b^2 - c^2} + \frac{bc}{b^2 + c^2 - a^2} + \frac{ca}{c^2 + a^2 - b^2} &= \sum_{cyc} \frac{ab}{a^2 + b^2 - c^2} = \\ &= \sum_{cyc} \frac{ab}{2ab \cos C} = \frac{1}{2} \sum_{cyc} \frac{1}{\cos C} = \frac{1}{2} \cdot \frac{3}{\cos\left(\frac{A+B+C}{3}\right)} = \frac{1}{2} \cdot \frac{3}{\cos\left(\frac{\pi}{3}\right)} = \frac{1}{2} \cdot \frac{3}{\frac{1}{2}} = 3 \end{aligned}$$

Equality holds for $a = b = c$.