ROMANIAN MATHEMATICAL MAGAZINE

If I −incenter in ΔABC then:

$$IA + IB + IC \leq 3R$$

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Solution by Daniel Sitaru-Romania

$$IA + IB + IC = \sum_{cyc} IA = \sum_{cyc} \frac{r}{\sin\frac{A}{2}} = r\sum_{cyc} \frac{1}{\sin\frac{A}{2}} \overset{JENSEN}{\leq}$$

$$\leq r \cdot \frac{3}{\sin\left(\frac{\frac{A}{2}+\frac{B}{2}+\frac{C}{2}}{3}\right)} = \frac{3r}{\sin\left(\frac{A+B+C}{6}\right)} = \frac{3r}{\sin\frac{\pi}{6}} \overset{EULER}{\leq} \frac{3\cdot\frac{R}{2}}{\frac{1}{2}} = 3R$$

Equality holds for an equilateral triangle.