ROMANIAN MATHEMATICAL MAGAZINE

If $n \in \mathbb{N}$ then in ΔABC the following relationship holds:

$$\frac{\cos\left(\frac{B-C}{2}\right)}{\sin^n\left(\frac{A}{2}\right)}+\frac{\cos\left(\frac{C-A}{2}\right)}{\sin^n\left(\frac{B}{2}\right)}+\frac{\cos\left(\frac{A-B}{2}\right)}{\sin^n\left(\frac{C}{2}\right)}\geq 3\cdot 2^n$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc}\frac{\cos\left(\frac{B-C}{2}\right)}{\sin^n\left(\frac{A}{2}\right)}=\sum_{cyc}\frac{\frac{b+c}{2}.\sin\left(\frac{A}{2}\right)}{\sin^n\left(\frac{A}{2}\right)}=\sum_{cyc}\frac{\frac{b+c}{2}}{\sin^{n-1}\left(\frac{A}{2}\right)}\overset{AM-GM}{\geq}\sum_{cyc}\frac{\frac{2\sqrt{bc}}{a}}{\sin^{n-1}\left(\frac{A}{2}\right)}\overset{AM-GM}{\geq}$$

$$\geq 3\left(\frac{\frac{2\sqrt{bc}}{a}.\frac{2\sqrt{ac}}{b}.\frac{2\sqrt{ba}}{c}}{\prod_{cyc}\sin^{n-1}\left(\frac{A}{2}\right)}\right)^{\frac{1}{3}}=3\left(\frac{8}{\left(\frac{r}{4R}\right)^{n-1}}\right)^{\frac{1}{3}}=$$

$$=3\left(8.4^{n-1}.\left(\frac{R}{r}\right)^{n-1}\right)^{\frac{1}{3}}\overset{Euler}{\geq}3(8.4^{n-1}.2^{n-1})^{\frac{1}{3}}=3(2^{3n})^{\frac{1}{3}}=3.2^n$$

$$Equality\ \ holds\ \ for: A=B=C.$$