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In any ΔABC the following relationship holds :

$$\sum_{\text{cyc}} (w_a \sqrt{w_a}) \geq 3(3r)^{\frac{3}{2}}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\begin{aligned} \sum_{\text{cyc}} (w_a \sqrt{w_a}) &\stackrel{\text{Chebyshev}}{\geq} \frac{1}{3} \left(\sum_{\text{cyc}} w_a \right) \left(\sum_{\text{cyc}} \sqrt{w_a} \right) \geq \frac{1}{3} \left(\sum_{\text{cyc}} h_a \right) \left(\sum_{\text{cyc}} \sqrt{h_a} \right) \\ &\stackrel{\text{Bergstrom and AM-GM}}{\geq} \frac{1}{3} \left(\frac{2rs \cdot 9}{2s} \right) \cdot 3 \cdot \sqrt[6]{\frac{2r^2 s^2}{R}} \stackrel{\text{Gerretsen + Euler}}{\geq} 3r \cdot 3 \cdot \sqrt[6]{\frac{r^2 \cdot 27Rr}{R}} = 9r \cdot \sqrt{3r} = 3(3r)^{\frac{3}{2}} \\ \therefore \sum_{\text{cyc}} (w_a \sqrt{w_a}) &\geq 3(3r)^{\frac{3}{2}} \forall \Delta ABC, '' = '' \text{ iff } \Delta ABC \text{ is equilateral (QED)} \end{aligned}$$