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In $\triangle ABC$ the following relationship holds:

$$\sum \frac{m_a}{a} \cdot \sum \frac{a}{\sqrt{r_b r_c}} \geq 9$$

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Solution by Tapas Das-India

$$m_a \geq \sqrt{s(s-a)} = \sqrt{\frac{s(s-a)(s-b)(s-c)}{(s-b)(s-c)}} = \frac{F}{\sqrt{(s-b)(s-c)}} = \sqrt{r_b r_c}$$

$$\sum \frac{m_a}{a} \cdot \sum \frac{a}{\sqrt{r_b r_c}} \geq \sum \frac{\sqrt{r_b r_c}}{a} \cdot \sum \frac{a}{\sqrt{r_b r_c}} \stackrel{C-S}{\geq} (1+1+1)^2 = 9$$

Equality holds for an equilateral triangle.