ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum_{cyc}\frac{h_a}{h_a^2+3rh_a+9r^2}\le\frac{1}{3r}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{h_a}{h_a^2+3rh_a+9r^2}=\frac{1}{h_a+3r+\frac{9r^2}{h_a}}=$$

$$=\frac{1}{3r+\left(h_a+\frac{9r^2}{h_a}\right)}\overset{AM-GM}{\le}\frac{1}{3r+2\sqrt{h_a\cdot\frac{9r^2}{h_a}}}=\frac{1}{3r+6r}=\frac{1}{9r}$$

$$\sum_{cyc}\frac{h_a}{h_a^2+3rh_a+9r^2}\le 3\cdot\frac{1}{9r}=\frac{1}{3r}$$

Equality holds if the triangle is an equilateral one.