

# ROMANIAN MATHEMATICAL MAGAZINE

In  $\triangle ABC$  the following relationship holds:

$$36r \leq \sum \frac{r_a}{\sin \frac{A}{2}} \leq \frac{9R^2}{r}$$

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*Solution by Tapas Das-India*

$$\frac{r_a}{\sin \frac{A}{2}} = \frac{Fbc}{(s-a)(s-b)(s-c)} = bc \cdot \frac{rs}{sr^2} = \frac{bc}{r}$$

$$\sum \frac{r_a}{\sin \frac{A}{2}} = \frac{1}{r} \sum bc \leq \frac{1}{r} \sum a^2 \stackrel{\text{Leibniz}}{\leq} \frac{9R^2}{r}$$

$$\sum \frac{r_a}{\sin \frac{A}{2}} = \frac{1}{r} \sum bc \stackrel{\text{Gordon}}{\geq} \frac{1}{r} \cdot 4\sqrt{3} F \stackrel{\text{Mitrinovic}}{\geq} \frac{1}{r} 4\sqrt{3} \cdot r \cdot 3\sqrt{3}r = 36r$$

Equality holds for an equilateral triangle.