

# ROMANIAN MATHEMATICAL MAGAZINE

In  $\Delta ABC$  the following relationship holds:

$$36r \leq \sum \frac{r_a}{\sin \frac{A}{2}} \leq \frac{9R^2}{r}$$

*Proposed by Marin Chirciu-Romania*

*Solution by Tapas Das-India*

$$\frac{r_a}{\sin \frac{A}{2}} = \frac{Fbc}{(s-a)(s-b)(s-c)} = bc \cdot \frac{rs}{sr^2} = \frac{bc}{r}$$

$$\sum \frac{r_a}{\sin \frac{A}{2}} = \frac{1}{r} \sum bc \leq \frac{1}{r} \sum a^2 \stackrel{Leibniz}{\leq} \frac{9R^2}{r}$$

$$\sum \frac{r_a}{\sin \frac{A}{2}} = \frac{1}{r} \sum bc \stackrel{Gordon}{\geq} \frac{1}{r} \cdot 4\sqrt{3} F \stackrel{Mitrinovic}{\geq} \frac{1}{r} 4\sqrt{3} \cdot r \cdot 3\sqrt{3}r = 36r$$

Equality holds for an equilateral triangle.