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In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} \frac{\sin^2(A)}{\sin^2(B) + \sin^2(C)} \leq \frac{2R - r}{2r}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} \sum_{cyc} \frac{\sin^2(A)}{\sin^2(B) + \sin^2(C)} &= \sum_{cyc} \frac{a^2}{b^2 + c^2} \stackrel{AM-GM}{\geq} \frac{1}{2} \sum_{cyc} \frac{a^2}{bc} = \sum_{cyc} \frac{a^3}{2abc} = \\ &= \frac{2(s^3 - 3sr^2 - 6sRr)}{2 \cdot 4Rsr} = \frac{p^2 - 3r^2 - 6Rr}{4Rr} \stackrel{Gerretsen}{\geq} \\ &\leq \frac{4R^2 + 4Rr + 3r^2 - 3r^2 - 6Rr}{4Rr} = \frac{4R^2 - 2Rr}{4Rr} = \frac{2R - r}{2r} \end{aligned}$$

Equality holds for: $A = B = C$.