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In ΔABC the following relationship holds:

$$\left(\sum_{cyc} r_a\right)\left(\sum_{cyc} \frac{1}{\cos^2\left(\frac{A}{2}\right)}\right) \leq \frac{9R^2}{r}$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$\left(\sum_{cyc} r_a\right)\left(\sum_{cyc} \frac{1}{\cos^2\left(\frac{A}{2}\right)}\right) = (4R+r)\left(1+\frac{(4R+r)^2}{s^2}\right)$$

$$\overset{Gerretsen}{\leq} (4R+r)\left(1+\frac{(4R+r)^2}{3r(4R+r)}\right) = (4R+r)\left(1+\frac{4R+r}{3r}\right) =$$

$$= (4R+r)\left(\frac{4(R+r)}{3r}\right) \overset{Euler}{\leq} \frac{9R}{2}\cdot\frac{(4R+2R)}{3r} = \frac{9R^2}{r}$$

$$Equality\ holds\ for\ A=B=C.$$