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In acute ΔABC the following relationship holds:

$$\frac{a^2}{\cos(A)}+\frac{b^2}{\cos(B)}+\frac{c^2}{\cos(C)}\geq 72r^2$$

Proposed by Gheorghe Crăciun-Romania

Solution 1 by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc}\frac{a^2}{\cos(A)}\overset{AM-GM}{\geq}3\left(\frac{(abc)^2}{\prod_{cyc}\cos(A)}\right)^{\frac{1}{3}}=$$

$$=3\left(\frac{16R^2F^2}{\prod_{cyc}\cos(A)}\right)^{\frac{1}{3}}\overset{Euler}{\geq}3(8.16.4r^2.p^2r^2)^{\frac{1}{3}}\overset{Mitrinovic}{\geq}3(8.16.27r^2.r^2.r^2)^{\frac{1}{3}}=72r^2$$

$$Equality\ holds\ for\ an\ equilateral\ triangle$$

Solution 2 by Chew Cheong-Malaysia

$$(\cos(A)+\cos(B)+\cos(C))\sum_{cyc}\frac{a^2}{\cos(A)}\geq(a+b+c)^2$$

$$\sum_{cyc}\frac{a^2}{\cos(A)}\geq\frac{(a+b+c)^2}{(\cos(A)+\cos(B)+\cos(C))}\geq\frac{2(a+b+c)^2}{3}=\frac{2(a+b+c)^3}{3(a+b+c)}\geq$$

$$\geq\frac{2(27abc)}{3(a+b+c)}=\frac{36Rabc}{2R(a+b+c)}=36Rr\geq 72r^2$$

$$Equality\ holds\ when\ A=B=C=60^\circ$$

$$Note:\cos(A)+\cos(B)+\cos(C)\leq\frac{3}{2}$$

$$Euler's\ inequality:\ R\geq 2r$$