ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC the following relationship holds :

$$\frac{a}{13a+13b+2c}+\frac{b}{13b+13c+2a}+\frac{c}{13c+13a+2b}\le\frac{3}{28}$$

Proposed by Dang Ngoc Minh-Vietnam

Solution by Soumava Chakraborty-Kolkata-India

$$\frac{a}{13a+13b+2c}+\frac{b}{13b+13c+2a}+\frac{c}{13c+13a+2b}\overset{?}{\le}\frac{3}{28}$$

$$\Leftrightarrow\sum_{cyc}\frac{(y+z)\big(13(z+x)+13(x+y)+2(y+z)\big)\big(13(x+y)+13(y+z)+2(z+x)\big)}{\prod_{cyc}\big(13(y+z)+13(z+x)+2(x+y)\big)}\overset{?}{\le}\frac{3}{28}$$

$$(x=s-a,y=s-b,z=s-c\Rightarrow a=y+z,b=z+x,c=x+y)$$

$$\Leftrightarrow 30\sum_{cyc}x^3+156\sum_{cyc}x^2y-143\sum_{cyc}xy^2-156xyz\ \boxed{\underset{(*)}{\overset{?}{\ge}}}\ 0$$

$$\text{Let } F(X,Y,Z)=30\sum_{cYc}X^3+156\sum_{cYc}X^2Y-143\sum_{cYc}XY^2-156XYZ\ \forall\ X,Y,Z\ge 0 \text{ and}$$

$$\text{then}: F(1,1,1)=0\to (1) \text{ and } F(X,Y,0)=30(X^3+Y^3)+165X^2Y-143XY^2$$

$$=\frac{(330X+2075Y)(33X-13Y)^2+11583XY^2+8695Y^3}{11979}\ge 0\because X,Y\ge 0$$

$$\Rightarrow F(X,Y,0)\ge 0\ \forall\ X,Y\ge 0\to (2)\ \therefore (1) \text{ and } (2)\Rightarrow F(X,Y,Z)\ge 0\ \forall\ X,Y,Z\ge 0$$

$$\Rightarrow (*) \text{ is true } (\because x,y,z>0) \text{ and so,}$$

$$\frac{a}{13a+13b+2c}+\frac{b}{13b+13c+2a}+\frac{c}{13c+13a+2b}\le\frac{3}{28}\ \forall\ ABC,$$

$$''=''\text{ iff } \Delta \text{ ABC is equilateral (QED)}$$