ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC the following relationship holds :

$$\left(\frac{s}{r}+\sqrt{2}\sum_{cyc}\sqrt{\frac{r_a}{h_a}}\right)\left(s\sum_{cyc}h_a-\sqrt{2}\sum_{cyc}\left(h_a\sqrt{r_ah_a}\right)\right)\geq\left(\sum_{cyc}n_a\right)^2$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\left(\frac{s}{r}+\sqrt{2}\sum_{cyc}\sqrt{\frac{r_a}{h_a}}\right)\left(s\sum_{cyc}h_a-\sqrt{2}\sum_{cyc}\left(h_a\sqrt{r_ah_a}\right)\right)=$$

$$=\left(s\sum_{cyc}\frac{1}{h_a}+\sum_{cyc}\frac{\sqrt{2r_ah_a}}{h_a}\right)\left(s\sum_{cyc}h_a-\sum_{cyc}\left(h_a\sqrt{2r_ah_a}\right)\right)=$$

$$=\left(\sum_{cyc}\left(\frac{1}{h_a}\left(s+\sqrt{2r_ah_a}\right)\right)\right).\left(\sum_{cyc}\left(h_a\left(s-\sqrt{2r_ah_a}\right)\right)\right)\overset{\text{Reverse CBS}}{\geq}$$

$$\geq\left(\sum_{cyc}\sqrt{\frac{1}{h_a},h_a.\left(s+\sqrt{2r_ah_a}\right)\left(s-\sqrt{2r_ah_a}\right)}\right)^2=\left(\sum_{cyc}\sqrt{\left(s^2-2r_ah_a\right)}\right)^2\overset{\text{Bogdan Fustei}}{=}$$

$$=\left(\sum_{cyc}\sqrt{n_a^2}\right)^2$$

$$\therefore\left(\frac{s}{r}+\sqrt{2}\sum_{cyc}\sqrt{\frac{r_a}{h_a}}\right)\left(s\sum_{cyc}h_a-\sqrt{2}\sum_{cyc}\left(h_a\sqrt{r_ah_a}\right)\right)\geq\left(\sum_{cyc}n_a\right)^2$$

$\forall\ \Delta$ ABC, $''=''$ iff Δ ABC is equilateral (QED)