ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \frac{h_a}{(s-a)^2 - \left(n_a - \sqrt{4r^2 + (b-c)^2}\right)^2} = \frac{4R + r}{2r^2}$$

Proposed by Bogdan Fuștei-Romania

Solution by Tapas Das-India

$$\frac{n_a}{h_a} = \frac{\sqrt{4r^2 + (b-c)^2}}{2r}, n_a^2 = s^2 - 2r_a h_a$$

$$(\textit{Refference}: \textit{Bogdan Fustei}$$
$$- \textit{ABOUT NAGEL AND GERGONNE CEVIANS}, www.ssmrmh.ro)$$

$$\textit{Known results}: \sum \frac{1}{s-a} = \frac{4R+r}{sr}, r_a = \frac{F}{s-a}$$

$$n_a - \sqrt{4r^2 + (b-c)^2} = n_a - \frac{n_a}{h_a}.2r = n_a\left(1 - \frac{a}{s}\right) = \frac{n_a(s-a)}{s}$$

$$(s-a)^2 - \left(n_a - \sqrt{4r^2 + (b-c)^2}\right)^2 = (s-a)^2 - \frac{n_a^2(s-a)^2}{s^2} =$$

$$= \frac{(s-a)^2}{s^2}(s^2 - n_a^2) = \frac{(s-a)^2}{s^2} 2r_a h_a$$

$$\frac{h_a}{(s-a)^2 - \left(n_a - \sqrt{4r^2 + (b-c)^2}\right)^2} = \frac{h_a}{\frac{(s-a)^2}{s^2} 2r_a h_a} = \frac{s^2}{2} \frac{1}{(s-a)^2 r_a} = \frac{s^2}{2F} \frac{.1}{s-a}$$

$$\sum \frac{h_a}{(s-a)^2 - \left(n_a - \sqrt{4r^2 + (b-c)^2}\right)^2} = \frac{s^2}{2F} \sum \frac{1}{s-a} = \frac{s}{2r} \frac{4R+r}{sr} = \frac{4R+r}{2r^2}$$