ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0,\ ab+bc+ca=3$ then:

$$\frac{a}{a^2+7}+\frac{b}{b^2+7}+\frac{c}{c^2+7}\le\frac{3}{8}$$

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Solution by Qurban Mueelim-Azerbaijan

$$LHS=\sum\frac{a}{a^2+7}=\sum\frac{a}{a^2+3+4}=\sum\frac{a}{a^2+ab+bc+ca+4}=$$

$$=\sum\frac{a}{(a+c)(a+b)+4}\le\sum\frac{a}{4\sqrt{(a+b)(a+c)}}=\frac{1}{8}\sum\frac{2a}{\sqrt{(a+b)(a+c)}}\le$$

$$\le\frac{1}{8}\cdot\sum\left(\frac{a}{a+b}+\frac{a}{a+c}\right)=\frac{1}{8}\cdot\sum\frac{a+b}{a+b}=\frac{1}{8}\cdot 3=\frac{3}{8}$$

$Equality\ holds\ for:\ a=b=c=1.$