

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$, $a + b + c = 3$ then:

$$\sum \frac{a^3}{a^2 + b^2} \geq \frac{3}{2}$$

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Solution by Tapas Das-India

$$\begin{aligned} \sum \frac{a^3}{a^2 + b^2} &= \sum a \cdot \frac{a^2}{a^2 + b^2} = \sum a \left(1 - \frac{b^2}{a^2 + b^2} \right) \stackrel{AM-GM}{\geq} \\ &\geq \sum a \left(1 - \frac{b^2}{2ab} \right) = \sum a - \frac{1}{2} \sum a = 3 - \frac{3}{2} = \frac{3}{2} \end{aligned}$$

Equality holds for $a=b=c=1$.