ROMANIAN MATHEMATICAL MAGAZINE

If $x,y,z>0,\ x+y+z+2=xyz$

$$\frac{1}{\sqrt{x+4}}+\frac{1}{\sqrt{y+4}}+\frac{1}{\sqrt{z+4}}\le\sqrt{\frac{3}{2}}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$x+y+z+2=xyz\ \ can\ be\ written\ as\ \frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}=1$$

$$\frac{1}{\sqrt{x+4}}+\frac{1}{\sqrt{y+4}}+\frac{1}{\sqrt{z+4}}\overset{CBS}{\le}\sqrt{3\left(\frac{1}{x+4}+\frac{1}{y+4}+\frac{1}{z+4}\right)}\le$$

$$=\sqrt{3\left(\frac{1}{(x+1)+3}+\frac{1}{(y+1)+3}+\frac{1}{(z+1)+3}\right)}=\sqrt{3\sum\frac{1}{(x+1)+3}}\overset{AM-HM}{\le}$$

$$\le\sqrt{\frac{3}{4}\sum\left(\frac{1}{x+1}+\frac{1}{3}\right)}=\sqrt{\frac{3}{4}\left(\sum\frac{1}{x+1}+\sum\frac{1}{3}\right)}=\sqrt{\frac{3}{4}(1+1)}=\sqrt{\frac{3}{2}}$$

Equality holds for $x=y=z=1$.