ROMANIAN MATHEMATICAL MAGAZINE

If $a,b>0$ then:

$$\frac{2}{a+b}+\frac{6}{4a+b+3}+\frac{6}{4b+a+3}\le\frac{1}{a}+\frac{1}{b}+\frac{3}{a+b+3}$$

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*Solution by Tapas Das-India*

$$\frac{2}{a+b}\overset{AM-HM}{\le}\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}\right)\ (1)$$

$$\frac{6}{4a+b+3}=\frac{6}{3a+(a+b+3)}=\frac{6}{4}\frac{4}{3a+(a+b+3)}\overset{AM-HM}{\le}\frac{3}{2}\left(\frac{1}{3a}+\frac{1}{a+b+3}\right)\ (2)$$

$$\frac{6}{4b+a+3}=\frac{6}{3b+(a+b+3)}=\frac{6}{4}\frac{4}{3b+(a+b+3)}\overset{AM-HM}{\le}\frac{3}{2}\left(\frac{1}{3b}+\frac{1}{a+b+3}\right)\ (3)$$

$$\frac{2}{a+b}+\frac{6}{4a+b+3}+\frac{6}{4b+a+3}\overset{(1),(2),(3)}{\le}$$

$$\le\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}\right)+\frac{3}{2}\left(\frac{1}{3a}+\frac{1}{a+b+3}\right)+\frac{3}{2}\left(\frac{1}{3b}+\frac{1}{a+b+3}\right)=\frac{1}{a}+\frac{1}{b}+\frac{3}{a+b+3}$$

Equality holds for a=b=1.