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If $a, \mathrm{b}, \mathrm{c} > 0$ and $\mathrm{n} \in \mathbb{N}^*$ then :

$$\sum_{cyc} \frac{c^{2n-2}}{a^{2n+1} + b^{2n+1} + abc^{2n-1}} \leq \frac{1}{abc}$$

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$$\sum_{cyc} \frac{c^{2n-2}}{a^{2n+1} + b^{2n+1} + abc^{2n-1}} \overset{\text{Chebyshev}}{\leq}$$

$$\leq \sum_{cyc} \frac{c^{2n-2}}{\frac{1}{2}(a^2 + b^2)(a^{2n-1} + b^{2n-1}) + abc^{2n-1}} \ (\because 2n - 1 \geq 1 \text{ as } n \in \mathbb{N}^*) \overset{\text{AM-GM}}{\leq}$$

$$\leq \sum_{cyc} \frac{c^{2n-2}}{\frac{1}{2}(2ab)(a^{2n-1} + b^{2n-1}) + abc^{2n-1}} = \sum_{cyc} \frac{c^{2n-2}}{ab(a^{2n-1} + b^{2n-1} + c^{2n-1})} =$$

$$= \sum_{cyc} \frac{c^{2n-1}}{abc(a^{2n-1} + b^{2n-1} + c^{2n-1})} = \frac{1}{abc} \cdot \frac{c^{2n-1} + a^{2n-1} + b^{2n-1}}{a^{2n-1} + b^{2n-1} + c^{2n-1}} = \frac{1}{abc}$$

$$\text{and so, } \sum_{cyc} \frac{c^{2n-2}}{a^{2n+1} + b^{2n+1} + abc^{2n-1}} \leq \frac{1}{abc} \ \forall a, b, c > 0 \text{ and } n \in \mathbb{N}^*,$$

$$'' ='' \text{ iff } a = b = c \ (\text{QED})$$