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If $a, b, c > 0$, $\lambda \geq 1$ then:

$$\sum \frac{a}{a+\lambda(b+c)} \geq \frac{3}{2\lambda+1}$$

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$$Let\ m = \sum a^2, n = \sum ab \ \ then\ m \geq n\ (1)\ (a+b+c)^2 = m + 2n\ (2)$$

$$\sum \frac{a}{a+\lambda(b+c)} = \sum \frac{a^2}{a^2+\lambda(ab+ac)} \overset{Bergstrom}{\geq} \frac{(a+b+c)^2}{\sum a^2 + 2\lambda \sum ab} \overset{(1)}{=} \frac{m+2n}{m+2\lambda n}$$

$$We\ need\ to\ show:$$

$$\frac{m+2n}{m+2\lambda n} \geq \frac{3}{2\lambda+1}\ or, 2\lambda m + 2n \geq 2m + 2\lambda n\ or, (\lambda-1)(m-n) \geq 0$$

$$true\ as\ \lambda \geq 1\ and\ m \geq n\ \big(by\ (1)\big)$$

Equality holds for a=b=c.