ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 3, abc = 64$ *then:*

$$\sum \frac{a^n}{a-3} \ge 3.4^n$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$Since\ a, b, c > 3\ , so\ we\ take\ a = x + 3, b = y + 3, z = c + 3\ , x, y, z > 0$$

$$abc = 64 \Rightarrow (x+3)(y+3)(z+3) = 64\ (1)$$

$$(x+3)(y+3)(z+3) = 64$$

$$(x+1+1+1)(y+1+1+1)(z+1+1+1) = 64$$

$$4\sqrt[4]{x}.4\sqrt[4]{y}.4\sqrt[4]{z}. \overset{AM-GM}{\le} 64\ or\ xyz \le 1\ (2)$$

$$\sum \frac{a^n}{a-3} = \sum \frac{(x+3)^n}{x} \overset{AM-GM}{\ge} 3\sqrt[3]{\frac{\left((x+3)(y+3)(z+3)\right)^n}{xyz}} \overset{(2)\&(1)}{\ge} 3.\sqrt[3]{64^n} = 3.4^n$$

Equality holds for x=y=z=1 or a=b=c=4.