

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0, a + b + c = 3$ then:

$$\sum \frac{a^{n+1} + b^n + c}{a^{n+2} + b^{n+1} + c^2} \leq 3, n \in \mathbb{N}$$

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Solution by Tapas Das-India

$$a^{n+2} + b^{n+1} + c^2 = a^{n+1} \cdot a + b^n \cdot b + c \cdot c \stackrel{\text{Chebyshev}}{\geq} \frac{1}{3}(a + b + c)(a^{n+1} + b^n + c)$$

$$\sum \frac{a^{n+1} + b^n + c}{a^{n+2} + b^{n+1} + c^2} \leq \sum \frac{a^{n+1} + b^n + c}{\frac{1}{3}(a + b + c)(a^{n+1} + b^n + c)} = 3 \sum \frac{1}{a + b + c} \stackrel{a+b+c=3}{=} 3$$

Equality holds for $a=b=c=1$.