ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$, $a^4 + b^4 + c^4 = 3$ then:

$$\frac{1}{a^2bc} + \frac{\lambda^2}{b^2ac} + \frac{(\lambda+1)^2}{c^2ab} \geq \frac{4(\lambda+1)^2 a^4b^4c^4}{a^4b^4 + b^4c^4 + c^4a^4}$$

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*Solution by Tapas Das-India*

$$a^4 + b^4 + c^4 = \frac{a^4}{1^3} + \frac{b^4}{1^3} + \frac{c^4}{1^3} \overset{Radon}{\geq} \frac{1}{27}(a+b+c)^4 =$$

$$= \left(\frac{a+b+c}{3}\right)^3 (a+b+c) \overset{AM-GM}{\geq} abc(a+b+c) = a^2bc + b^2ca + c^2ab \ (1)$$

$$a^4b^4 + b^4c^4 + c^4a^4 = a^4b^4c^4\left(\frac{1}{a^4} + \frac{1}{b^4} + \frac{1}{c^4}\right) \overset{Bergstrom}{\geq} a^4b^4c^4 \cdot \frac{9}{a^4 + b^4 + c^4} \ (2)$$

$$\frac{1}{a^2bc} + \frac{\lambda^2}{b^2ac} + \frac{(\lambda+1)^2}{c^2ab} \overset{Bergstrom}{\geq} \frac{(1+\lambda+\lambda+1)^2}{a^2bc + b^2ca + c^2ab} \overset{(1)}{\geq} \frac{4(\lambda+1)^2}{a^4 + b^4 + c^4} \ (3)$$

$$(a^4b^4 + b^4c^4 + c^4a^4)\left(\frac{1}{a^2bc} + \frac{\lambda^2}{b^2ac} + \frac{(\lambda+1)^2}{c^2ab}\right) \overset{(2)\&(3)}{\geq}$$

$$\geq a^4b^4c^4 \cdot \frac{9}{a^4 + b^4 + c^4} \cdot \frac{4(\lambda+1)^2}{a^4 + b^4 + c^4} \overset{a^4+b^4+c^4=3}{=} a^4b^4c^4 . 4(\lambda+1)^2$$

$$\frac{1}{a^2bc} + \frac{\lambda^2}{b^2ac} + \frac{(\lambda+1)^2}{c^2ab} \geq \frac{4(\lambda+1)^2 a^4b^4c^4}{a^4b^4 + b^4c^4 + c^4a^4}$$

Equality holds for a=b= c=1.