If $a, \mathrm{b} > 0, a + \mathrm{b} = 2$ and $\lambda \geq 2$ then :

$$\frac{1}{1 - a + \lambda a^2} + \frac{1}{1 - \mathrm{b} + \lambda \mathrm{b}^2} \geq \frac{2}{\lambda}$$

Proposed by Marin Chirciu-Romania

Solution by Soumava Chakraborty-Kolkata-India

$$2 = a + \mathrm{b} \overset{\mathrm{AM-GM}}{\geq} 2.\sqrt{a\mathrm{b}} \Rightarrow a\mathrm{b} \leq 1 \to (1)$$

Now, $\dfrac{1}{1 - a + \lambda a^2} + \dfrac{1}{1 - \mathrm{b} + \lambda \mathrm{b}^2} \overset{?}{\geq} \dfrac{2}{\lambda} \Leftrightarrow \dfrac{\lambda a^2 + \lambda \mathrm{b}^2}{(1 - a + \lambda a^2)(1 - \mathrm{b} + \lambda \mathrm{b}^2)} \overset{?}{\geq} \dfrac{2}{\lambda}$

$$(\because a + \mathrm{b} = 2) \Leftrightarrow \lambda^2(a^2 + \mathrm{b}^2 - 2a^2\mathrm{b}^2) + 2\lambda(2a\mathrm{b} - a^2 - \mathrm{b}^2) + 2 - 2a\mathrm{b} \underset{(*)}{\overset{?}{\geq}} 0$$

$(\because a + \mathrm{b} = 2)$ and indeed, via (1), LHS of $(*) \geq$

$$\lambda^2(a^2 + \mathrm{b}^2 - 2a\mathrm{b}) + 2\lambda(2a\mathrm{b} - a^2 - \mathrm{b}^2) + 2 - 2 = \lambda(\lambda - 2)(a - \mathrm{b})^2 \geq 0 (\because \lambda \geq 2)$$

$$\Rightarrow (*) \text{ is true } \therefore \frac{1}{1 - a + \lambda a^2} + \frac{1}{1 - \mathrm{b} + \lambda \mathrm{b}^2} \geq \frac{2}{\lambda}\ \forall a, \mathrm{b} > 0 \,\middle|\, a + \mathrm{b} = 2 \text{ and } \lambda \geq 2,$$

$'' ='' $ iff $a = \mathrm{b} = 1$ (QED)