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If $a, b, c > 0$ and $\lambda \geq 0$ then :

$$\sum_{\text{cyc}} \frac{b^2 c^2}{a^3 (b + \lambda c)} \geq \frac{3}{\lambda + 1}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\begin{aligned} \sum_{\text{cyc}} \frac{b^2 c^2}{a^3 (b + \lambda c)} &= \sum_{\text{cyc}} \frac{b^4 c^4}{a^3 b^2 c^2 (b + \lambda c)} \stackrel{\text{Bergstrom}}{\geq} \frac{(\sum_{\text{cyc}} a^2 b^2)^2}{a^2 b^2 c^2 (\lambda + 1) (\sum_{\text{cyc}} ab)} \geq \\ &\frac{3 a^2 b^2 c^2 (\sum_{\text{cyc}} a^2)}{a^2 b^2 c^2 (\lambda + 1) (\sum_{\text{cyc}} ab)} \geq \frac{3 (\sum_{\text{cyc}} ab)}{(\lambda + 1) (\sum_{\text{cyc}} ab)} = \frac{3}{\lambda + 1} \quad \forall a, b, c > 0 \wedge \lambda \geq 0, \end{aligned}$$

" = " iff $a = b = c$ (QED)