ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0,\ a^5 + b^5 + c^5 = 3$ then:

$$\frac{(a^5+1)^2}{b^3+b^2} + \frac{(b^5+1)^2}{c^3+c^2} + \frac{(c^5+1)^2}{a^3+a^2} \geq 6$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{(a^5+1)^2}{b^3+b^2} + \frac{(b^5+1)^2}{c^3+c^2} + \frac{(c^5+1)^2}{a^3+a^2} \overset{Bergstrom}{\geq} \frac{(a^5+b^5+c^5+3)^2}{\sum_{cyc}(a^3+a^2)} = \frac{(3+3)^2}{\sum_{cyc}(a^3+a^2)}$$

$$Let's\ \ see\ the\ difference:$$

$$a > 0$$

$$(a^5+1) - (a^3+a^2) = a^3(a^2-1) - (a^2-1) =$$

$$(a^2-1)(a^3-1) = (a-1)^2(a+1)(a^2+a+1) \geq 0\ \rightarrow a^3 + a^2 \leq a^5 + 1$$

$$\sum_{cyc} \frac{(a^5+1)^2}{b^3+b^2} \geq \frac{36}{\sum_{cyc}(a^3+a^2)} \geq \frac{36}{\sum_{cyc}(a^5+1)} = \frac{36}{3+3} = 6$$

$$Equality\ holds\ for: a = b = c = 1.$$