ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0$ then:

$$4(x+y+z)(x^2+y^2+z^2+xy+yz+zx) \geq 9(x+y)(y+z)(z+x)$$

Proposed by Gheorghe Crăciun-Romania

Solution by Tapas Das-India

$$4(x+y+z)(x^2+y^2+z^2+xy+yz+zx) =$$

$$= (2x+2y+2z)(2x^2+2y^2+2z^2+2xy+2yz+2zx) =$$

$$= \big((x+y)+(y+z)+(z+x)\big)\left((x+y)^2+(y+z)^2+(z+x)^2\right) \geq$$

$$\overset{AM-GM}{\geq} 3\sqrt[3]{(x+y)(y+z)(z+x)} \times 3\sqrt[3]{(x+y)^2(y+z)^2(z+x)^2} =$$

$$= 9(x+y)(y+z)(z+x)$$

Equality holds for x=y=z.