ROMANIAN MATHEMATICAL MAGAZINE

If $0 < a, b, c < 1$ then:

$$\frac{2a+abc}{bc+1}+\frac{2b+abc}{ac+1}+\frac{2c+abc}{ab+1}<3+ab+bc+ca$$

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$$0 < a, b, c < 1 \text{ so } a+b+c < 3 \ (1)$$

$$\frac{2a+abc}{bc+1}=\frac{2a}{1+bc}+\frac{abc}{1+bc}$$

$$\text{since } bc > 0 \text{ so } \frac{2a}{1+bc}<2a \text{ and } \frac{abc}{1+bc}<abc<ab$$

$$\left(\text{as } abc-ab=ab(c-1)\overset{c<1}{<}0 \Rightarrow abc<ab\right)$$

$$\frac{2a+abc}{bc+1}=\frac{2a}{1+bc}+\frac{abc}{1+bc}<2a+ab$$

$$\frac{2a+abc}{bc+1}+\frac{2b+abc}{ac+1}+\frac{2c+abc}{ab+1}=$$

$$=\sum\frac{2a+abc}{bc+1}<\sum 2a+\sum ab\overset{(1)}{<}2\times 3+ab+bc+ca=6+ab+bc+ca$$