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If $a, b, c, d > 0$ then:

$$\frac{a^2-bc+cd}{bc}+\frac{b^2-cd+bc}{cd}+\frac{c^2-ad+ba}{ad}+\frac{d^2-ab+ad}{ab}\geq 4$$

Proposed by Gheorghe Crăciun-Romania

Solution by Tapas Das-India

$$\frac{a^2-bc+cd}{bc}=\frac{a^2}{bc}-\frac{bc}{bc}+\frac{cd}{bc}=\frac{a^2}{bc}-1+\frac{d}{b}$$

$$\frac{a^2-bc+cd}{bc}+\frac{b^2-cd+bc}{cd}+\frac{c^2-ad+ba}{ad}+\frac{d^2-ab+ad}{ab}=\sum\frac{a^2-bc+cd}{bc}$$

$$=\sum\frac{a^2}{bc}+\sum\frac{d}{b}-4\overset{AM-GM}{\geq}4\sqrt[4]{\frac{a^2b^2d^2c^2}{a^2b^2d^2c^2}}+4\sqrt[4]{\frac{abcd}{abcd}}-4=4$$

Equality holds for a=b=c=d.