ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0,\ a + b + c = 1$ then:

$$\frac{3a}{4-3a}+\frac{3b}{4-3b}+\frac{3c}{4-3c}+a^4+b^4+c^4 \geq a+b+c+abc$$

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$$LHS\ (part-1):\left(\frac{3a}{4-3a}+1\right)+\left(\frac{3b}{4-3b}+1\right)+\left(\frac{3c}{4-3c}+1\right)$$

$$\frac{4}{4-3a}+\frac{4}{4-3b}+\frac{4}{4-3c}\ \underset{(1)}{\overset{Bergstrom}{\geq}}\ \frac{(2+2+2)^2}{12-3(a+b+c)}=\frac{36}{9}=4$$

$$\frac{3a}{4-3a}+\frac{3b}{4-3b}+\frac{3c}{4-3c}\geq 4-3=1=a+b+c\ (*)$$

$$LHS\ (part-2)\ a^4+b^4+c^4\ \underset{(2)}{\overset{Chebyshev}{\geq}}\ \frac{1}{3}(a+b+c)(a^3+b^3+c^3)=$$

$$\frac{1}{3}(a^3+b^3+c^3)\ \underset{(3)}{\overset{A-G}{\geq}}\ \frac{1}{3}.3abc=abc$$

$$a^4+b^4+c^4\geq abc\quad (**)$$

$$From\ (*)\ and\ (**)\ we\ get:$$

$$\frac{3a}{4-3a}+\frac{3b}{4-3b}+\frac{3c}{4-3c}+a^4+b^4+c^4\geq a+b+c+abc.$$

$$Equality\ holds\ for: a=b=c=\frac{1}{3}.$$