ROMANIAN MATHEMATICAL MAGAZINE

If $x, y \in (0, \pi]$ then:

$$\frac{2}{(\sin^2 x - \sin x + 1)(\sin^2 y - \sin y + 1)} \leq 1 + \frac{1}{\sin^2 x \sin^2 y}$$

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$$\sin^2 x - \sin x + 1 = (\sin^2 x + 1) - \sin x \overset{AM-GM}{\geq} 2\sin x - \sin x = \sin x$$

$$\textit{Similarly}:$$

$$\sin^2 y - \sin y + 1 \geq \sin y \ \textit{ then } (\sin^2 x - \sin x + 1)(\sin^2 y - \sin y + 1) \geq \sin x \sin y$$

$$\textit{We need to show}:$$

$$\frac{2}{(\sin^2 x - \sin x + 1)(\sin^2 y - \sin y + 1)} \leq 1 + \frac{1}{\sin^2 x \sin^2 y}$$

$$\frac{2}{\sin x \sin y} \leq 1 + \frac{1}{\sin^2 x \sin^2 y}$$

$$1 + \frac{1}{\sin^2 x \sin^2 y} - \frac{2}{\sin x \sin y} \geq 0$$

$$\left(1 - \frac{1}{\sin x \sin y}\right)^2 \geq 0 \ \textit{true}$$

$$\textit{Equality holds for } x = y = \frac{\pi}{2}.$$