ROMANIAN MATHEMATICAL MAGAZINE

JP. 581 If $x, y, z \geq 0$ then:

$$\frac{[3^x]\cdot[3^y]}{[3^{x+y}]}+\frac{[3^y]\cdot[3^z]}{[3^{y+z}]}+\frac{[3^z]\cdot[3^x]}{[3^{z+x}]}\leq 3$$

$[*]$ - great integer function.

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Solution by proposer

Lemma: If $a, b \in [0, \infty)$ then:

$$[a]\cdot[b]\leq[a\cdot b] \quad (1)$$

Proof:

$$[a]\leq a; [b]\leq b \Rightarrow [a]\cdot[b]\leq ab$$

$$\Rightarrow \big[[a]\cdot[b]\big]\leq[a\cdot b]\Rightarrow[a]\cdot[b]\leq[a\cdot b]$$

Back to the problem:

We take in (1): $a = 3^x; b = 3^y$

$$[3^x]\cdot[3^y]\leq[3^x\cdot 3^y]\Rightarrow\frac{[3^x]\cdot[3^y]}{[3^{x+y}]}\leq 1 \quad (2)$$

Analogous:

$$\frac{[3^y]\cdot[3^z]}{[3^{y+z}]}\leq 1 \quad (3)$$

$$\frac{[3^z]\cdot[3^x]}{[3^{z+x}]}\leq 1 \quad (4)$$

By adding (2); (3); (4):

$$\frac{[3^x]\cdot[3^y]}{[3^{x+y}]}+\frac{[3^y]\cdot[3^z]}{[3^{y+z}]}+\frac{[3^z]\cdot[3^x]}{[3^{z+x}]}\leq 3$$

Equality holds for $x, y, z \in \mathbb{N}$.