ROMANIAN MATHEMATICAL MAGAZINE

JP.580 If $a,b\geq 0;x\in\left(0,\frac{\pi}{2}\right)$ then:

$$\frac{[2^a+\sin^2 x]+[2^b+\cos^2 x]}{[2^a+2^b]}+\frac{[2^b+\sin^2 x]+[2^c+\cos^2 x]}{[2^b+2^c]}+\frac{[2^c+\sin^2 x]+[2^a+\cos^2 x]}{[2^c+2^a]}\geq 3$$

[*] – great integer function.

Proposed by Daniel Sitaru – Romania

Solution by proposer

$$2^a+\sin^2 x-1<[2^a+\sin^2 x]$$
$$2^b+\cos^2 x-1<[2^b+\cos^2 x]$$

By adding:

$$2^a+2^b+\sin^2 x+\cos^2 x-2<[2^a+\sin^2 x]+[2^b+\cos^2 x]$$
$$2^a+2^b+1-2<[2^a+\sin^2 x]+[2^b+\cos^2 x]$$
$$2^a+2^b-1<[2^a+\sin^2 x]+[2^b+\cos^2 x]$$
$$[2^a+2^b-1]<\big[[2^a+\sin^2 x]+[2^b+\cos^2 x]\big]$$
$$[2^a+2^b]\leq[2^a+\sin^2 x]+[2^b+\cos^2 x]$$
$$\frac{[2^a+\sin^2 x]+[2^b+\cos^2 x]}{[2^a+2^b]}\geq 1 \qquad (1)$$

Analogous:

$$\frac{[2^b+\sin^2 x]+[2^c+\cos^2 x]}{[2^b+2^c]}\geq 1 \qquad (2)$$
$$\frac{[2^c+\sin^2 x]+[2^a+\cos^2 x]}{[2^c+2^a]}\geq 1 \qquad (3)$$

By adding (1); (2); (3):

$$\frac{[2^a+\sin^2 x]+[2^b+\cos^2 x]}{[2^a+2^b]}+\frac{[2^b+\sin^2 x]+[2^c+\cos^2 x]}{[2^b+2^c]}+$$
$$+\frac{[2^c+\sin^2 x]+[2^a+\cos^2 x]}{[2^c+2^a]}\geq 3$$