ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{m_a^2+m_b^2}{m_am_b+m_c}+\frac{m_c^2+m_b^2}{m_cm_b+m_a}+\frac{m_a^2+m_c^2}{m_am_c+m_b}\ge\frac{36r}{3R+2}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{m_a^2}{m_am_b+m_c}+\frac{m_b^2}{m_am_b+m_c}+\frac{m_c^2}{m_cm_b+m_a}=$$

$$=\frac{m_b^2}{m_cm_b+m_a}+\frac{m_a^2}{m_am_c+m_b}+\frac{m_c^2}{m_am_c+m_b}\overset{Bergstrom}{\ge}$$

$$\ge\frac{(2m_a+2m_b+2m_c)^2}{2(m_am_b+m_cm_b+m_am_c)+2(m_a+m_b+m_c)}=$$

$$=\frac{2(m_a+m_b+m_c)^2}{(m_am_b+m_cm_b+m_am_c)+(m_a+m_b+m_c)}=$$

$$=\frac{6(m_a+m_b+m_c)^2}{3(m_am_b+m_cm_b+m_am_c)+3\sum_{cyc}m_a}\ge$$

$$\ge\frac{6(m_a+m_b+m_c)^2}{(m_a+m_b+m_c)^2+3(m_a+m_b+m_c)}=$$

$$=\frac{6(m_a+m_b+m_c)}{(m_a+m_b+m_c)+3}\overset{Gotman-2}{\ge}\frac{6(m_a+m_b+m_c)}{\frac{9R}{2}+3}=$$

$$=\frac{12(m_a+m_b+m_c)}{9R+6}=\frac{4(m_a+m_b+m_c)}{3R+2}\overset{Gotman-1}{\ge}\frac{36r}{3R+2}$$

Equality holds for an equilateral triangle.