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If $n\in\mathbb{N}, n\ge 2$ then in ΔABC the following relationship holds:

$$\prod_{cyc}\cos\frac{A-B}{2}\ge\left(8\prod_{cyc}\sin\frac{A}{2}\right)^n$$

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Solution by Daniel Sitaru-Romania

$$\prod_{cyc}\cos\frac{A-B}{2}\ge\left(8\prod_{cyc}\sin\frac{A}{2}\right)^n\Leftrightarrow\frac{s^2+r^2+2Rr}{8R^2}\ge\left(8\cdot\frac{r}{4R}\right)^n\Leftrightarrow$$

$$\Leftrightarrow\frac{s^2+r^2+2Rr}{8R^2}\ge\frac{2^n\cdot r^n}{R^n}\Leftrightarrow s^2+r^2+2Rr\ge\frac{2^{n+3}\cdot r^n}{R^{n-2}}$$

$$s^2+r^2+2Rr\overset{MITRINOVICI}{\ge}27r^2+r^2+2Rr\overset{EULER}{\ge}28r^2+4r^2=2^5r^2$$

Remains to prove:

$$2^5r^2\ge\frac{2^{n+3}\cdot r^n}{R^{n-2}}\Leftrightarrow R^{n-2}\ge 2^{n-2}\cdot r^{n-2}\Leftrightarrow R\overset{EULER}{\ge}2r$$

Equality holds for $a=b=c$.