

ROMANIAN MATHEMATICAL MAGAZINE

If I –incenter in $\triangle ABC$ then:

$$\frac{abc}{IA \cdot IB \cdot IC} \geq 3\sqrt{3}$$

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Solution by Daniel Sitaru-Romania

$$\begin{aligned} \frac{abc}{IA \cdot IB \cdot IC} &= \frac{4RF}{\frac{r}{\sin \frac{A}{2}} \cdot \frac{r}{\sin \frac{B}{2}} \cdot \frac{r}{\sin \frac{C}{2}}} = \frac{4Rrs}{r^3} \cdot \prod_{cyc} \sin \frac{A}{2} = \\ &= \frac{4Rs}{r^2} \cdot \frac{r}{4R} = \frac{s}{r} \stackrel{MITRINOVICI}{\geq} \frac{3\sqrt{3}r}{r} = 3\sqrt{3} \end{aligned}$$

Equality holds for $a = b = c$.