ROMANIAN MATHEMATICAL MAGAZINE

If I –incenter in ΔABC then:

$$\frac{1}{IA}+\frac{1}{IB}+\frac{1}{IC}\leq\frac{3}{2r}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\frac{1}{IA}+\frac{1}{IB}+\frac{1}{IC}=\sum_{cyc}\frac{1}{IA}=\sum_{cyc}\frac{1}{\frac{r}{\sin\frac{A}{2}}}=\frac{1}{r}\sum_{cyc}\sin\frac{A}{2}\leq$$

$$\overset{JENSEN}{\leq}\frac{1}{r}\cdot 3\sin\left(\frac{\frac{A}{2}+\frac{B}{2}+\frac{C}{2}}{3}\right)=\frac{3}{r}\sin\frac{A+B+C}{6}=\frac{3}{r}\sin\frac{\pi}{6}=\frac{3}{2r}$$

Equality holds for $a=b=c$.