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In ΔABC the following relationship holds:

$$(a^2)^{tan\frac{A}{2}}+(b^2)^{tan\frac{B}{2}}+(c^2)^{tan\frac{C}{2}}\ge 3\left(\frac{4F\sqrt{3}}{3}\right)^{\frac{\sqrt{3}}{3}}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc}(a^2)^{tan\frac{A}{2}}\overset{A-G}{\ge}3\left(\prod_{cyc}(a^2)^{tan\frac{A}{2}}\right)^{\frac{1}{3}}=$$

$$=3\left(\left((a^2)^{tan\frac{A}{2}}\right).\left((b^2)^{tan\frac{B}{2}}\right).\left((c^2)^{tan\frac{C}{2}}\right)\right)^{\frac{1}{3}}\overset{Weighted\ \ GM\ge HM}{\ge}$$

$$3\left(\frac{tan\frac{A}{2}+tan\frac{B}{2}+tan\frac{C}{2}}{\frac{tan\frac{A}{2}}{a^2}+\frac{tan\frac{B}{2}}{b^2}+\frac{tan\frac{C}{2}}{c^2}}\right)^{\frac{1}{3}\cdot\sum tan\frac{A}{2}}\qquad\sum_{cyc}\frac{tan\frac{A}{2}}{a^2}\overset{Chebyshev}{\le}\frac{1}{3}\sum_{cyc}\frac{1}{a^2}\cdot\sum_{cyc}tan\frac{A}{2};$$

$$WLOG:\ a\le b\le c,\qquad \frac{1}{a^2}\ge\frac{1}{b^2}\ge\frac{1}{c^2},\qquad tan\frac{A}{2}\le tan\frac{B}{2}\le tan\frac{C}{2}$$

$$\ge 3\left(\frac{\sum_{cyc}tan\frac{A}{2}}{\frac{1}{3}\sum_{cyc}\frac{1}{a^2}\cdot\sum_{cyc}tan\frac{A}{2}}\right)^{\frac{1}{3}\cdot\sum_{cyc}tan\frac{A}{2}}\overset{\sum_{cyc}tan\frac{A}{2}\ge\sqrt{3}}{\ge}3\left(\frac{3a^2b^2c^2}{a^2b^2+b^2c^2+a^2c^2}\right)^{\frac{1}{3}\cdot\sqrt{3}}\overset{Goldstone}{\ge}$$

$$3\left(\frac{3(4FR)^2}{4R^2S^2}\right)^{\frac{\sqrt{3}}{3}}=3\left(\frac{3.4.F^2}{S^2}\right)^{\frac{\sqrt{3}}{3}}=3\left(\frac{3F.4F}{S^2}\right)^{\frac{\sqrt{3}}{3}}\ge 3\left(\frac{4F\sqrt{3}}{3}\right)^{\frac{\sqrt{3}}{3}}$$

$$Equality\ \ holds\ \ for:\ a=b=c.$$

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