ROMANIAN MATHEMATICAL MAGAZINE

Prove that in any acute ΔABC:

$$(\tan A+\tan B)^{\tan A+\tan B}.(\tan C+\tan B)^{\tan C+\tan B}.(\tan A+\tan C)^{\tan A+\tan C}\ge\left(2\sqrt{3}\right)^{6\sqrt{3}}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Mirsadix Muzefferov-Azerbaijan

$$(\tan A+\tan B)^{\tan A+\tan B}.(\tan C+\tan B)^{\tan C+\tan B}.(\tan A+\tan C)^{\tan A+\tan C}=$$

$$=\prod_{cyc}(\tan A+\tan B)^{\tan A+\tan B}\overset{Weighted\ \ GM-HM}{\ge}\left(\frac{2.\sum_{cyc}\tan A}{\sum_{cyc}\frac{\tan A+\tan B}{\tan A+\tan B}}\right)^{2\sum_{cyc}\tan A}=$$

$$=\left(\frac{2.\sum_{cyc}\tan A}{1+1+1}\right)^{2\sum_{cyc}\tan A}\ge\left(\frac{2\cdot 3\sqrt{3}}{3}\right)^{2\cdot 3\sqrt{3}}=\left(2\sqrt{3}\right)^{6\sqrt{3}}$$

Equality holds for: $A=B=C$