ROMANIAN MATHEMATICAL MAGAZINE

In any acute ΔABC the following relationship holds:

$$\tan A+\tan B+\tan C\ge\frac{3(\sin A+\sin B+\sin C)}{\cos A+\cos B+\cos C}$$

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$$Known\ results:$$
$$1)\sum\sin A=\frac{s}{R}$$
$$2)\sum\cos A=1+\frac{r}{R},$$
$$3)\sum\tan A=\frac{2sr}{s^2-(2R+r)^2}$$

$$Using\ above\ results\ we\ need\ to\ show:$$

$$\tan A+\tan B+\tan C\ge\frac{3(\sin A+\sin B+\sin C)}{\cos A+\cos B+\cos C}$$
$$\frac{2sr}{s^2-(2R+r)^2}\ge\frac{3.\frac{s}{R}}{1+\frac{r}{R}}$$

$$2(R+r)r\ge 3s^2-3(2R+r)^2$$

$$2Rr+2r^2\overset{Gerretsen}{\ge}3(4R^2+4Rr+3r^2)-3(2R+r)^2$$

$$2Rr+2r^2\ge 6r^2\ or,R\ge 2r\ true\ by\ Euler$$

Equality holds for an equilateral triangle.