ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \cos A - 2\sum \cos A \cos A \geq 0$$

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$$\sum \cos A - 2\sum \cos A \cos A = \left(1+\frac{r}{R}\right) - 2\frac{s^2+r^2-4R^2}{4R^2} \geq$$

$$\overset{Gerretsen}{\geq} \left(1+\frac{r}{R}\right) - \frac{4R^2+4Rr+3r^2+r^2-4R^2}{2R^2} =$$

$$= 1+\frac{r}{R} - \frac{2r}{R} - 2\left(\frac{r}{R}\right)^2 \overset{\frac{r}{R}=x\leq\frac{1}{2}}{=} 1-x-2x^2$$

$$\textit{According to the problem we need to show}:$$

$$1-x-2x^2 \geq 0 \text{ or } 2x^2+x-1 \leq 0$$

$$(x+1)(2x-1) \leq 0 \text{ true as } x = \frac{r}{R} \leq \frac{1}{2} (Euler)$$

Equality holds for an equilateral triangle.