ROMANIAN MATHEMATICAL MAGAZINE

In acute ΔABC the following relationship holds:

$$\sin A\sin 2A+\sin B\sin 2B+\sin C\sin 2C\le \frac{9}{4}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\sin A\sin 2A+\sin B\sin 2B+\sin C\sin 2C=$$

$$=\sum \sin A\sin 2A=2\sum \sin^2 A\cos A\overset{Chebyshev}{\le}\frac{2}{3}\left(\sum \sin^2 A\right)\left(\sum \cos A\right)=$$

$$=\frac{2}{3}\frac{a^2+b^2+c^2}{4R^2}.\left(1+\frac{r}{R}\right)\overset{Leibniz\ \&\ Euler}{\le}\frac{2}{3}\cdot\frac{9R^2}{4R^2}\left(1+\frac{1}{2}\right)=\frac{9}{4}$$

Equality holds for an equilateral triangle.